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On the entropy-viscosity method for flux reconstruction

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1 Introduction

Recent advances in modern computer architectures for high-performance computing (HPC) are paving the path towards a wider adoption of high-order (HO) methods within the computational fluid dynamics (CFD) community (Castonguay, 2012). Compared to traditional low-order methods, HO methods promise to achieve an arbitrary level of accuracy at a reduced computational cost (Witherden, 2015), making high-fidelity scale-resolving simulations for high-speed flows a reality. Moreover, HO extensions of traditional methods are usually constructed through larger stencils – an approach tied to be memory-bounded in the new hardware architectures. In contrast, the element-local operations of HO methods helps reducing such memory overhead (Trojak, 2019) and is more aligned with the current HPC vision.

Under the high-order methods umbrella, the flux reconstruction (FR) method originally proposed by Huynh (2007) has been gaining attention due to its simple formulation and unifying framework. Similarly to other HO methods such as nodal discontinuous Galerkin (DG), the FR method considers piecewise discontinuous polynomial basis functions (usually Lagrange polynomials) defined at a set of element nodal points to spatially approximate the solution of a conservation law in a tessellated computational domain. The FR method is also linked to the spectral difference (SD) method in the sense that the differential form of the conservation law is used, differently from nodal DG which uses the integral weak form approach. It has been shown that FR can recover both nodal DG and SD schemes for linear and spatially-varying fluxes (Vincent, 2011), and non-linear fluxes (DeGrazia, 2014), hence the unifying character.

During the last decade, research on HO methods for CFD has been focused on achieving a similar level of maturity as traditional low-order methods. To do so, classical turbulence models, shock-capturing schemes, and convection schemes, among other key features, need to be revisited for the HO approach. With this purpose, we investigate the entropy-viscosity shock-capturing scheme by Guermond (2011) for the FR method. The entropy-viscosity method forces an entropy-based numerical dissipation via effective viscosity near physical discontinuities while vanishing on smooth regions. This helps stabilising the naturally arising oscillations that spectral (HO) methods trigger near discontinuities because of the Gibbs phenomenon. The main idea of this method is to capture the shocks occurring inside the element, while the discontinuities between elements are dealt by a Riemann solver. Because of the close relation of FR to nodal DG, shock-capturing schemes originally developed for DG can be translated to FR. In this context, Trojak (2021) combined a HO FR scheme with a low-order summation-by-parts scheme via convex limiting allowing to accurately capture shocks for the 1D Euler equations. The entropy-viscosity residual was used as shock sensor to lower the computational cost of the method. Asthana (2015) implemented a shock-capturing scheme based on Fourier spectral filtering which allowed to accurately capture shocks located inside an element even for high polynomial degrees.

2 Methodology and results

A 1-D flux reconstruction solver has been implemented using Julia (Bezanson, 2017): a compiled, dynamic, and composable programming language specifically designed for scientific computing. The solver features the energy-stable schemes from Vincent (2011), arbitrary polynomial degree for the Lagrange basis functions, Gauss–Legendre (GL) or Gauss–Lobatto–Legendre (GLL) collocation points, a set of different Riemann solvers for the numerical fluxes (Roe, Rusanov, HLL, HLLC), and low dissipation, low dispersion 4th order 2N Runge–Kutta temporal integration schemes (Stanescu, 1998). In contrast to other works, the implementation of the entropy-viscosity scheme is performed element-wise, *ie.* a single elemental viscosity is used taken as the maximum absolute norm of the values computed at the element solution points. This has proven to be more stable than the point-wise counterpart.

Encouraging preliminary results have been obtained for the 1D Burgers equations. This set of equations forms a shock when two or more characteristic lines intersect each other. Two initial conditions have been explored, a sine and a square wave. The solution is obtained using polynomials from third to fifth degree ($p \in \{3, 4, 5\}$) and two different quadratures (GL and GLL) with and without entropy viscosity. The numerical experiments show that the entropy

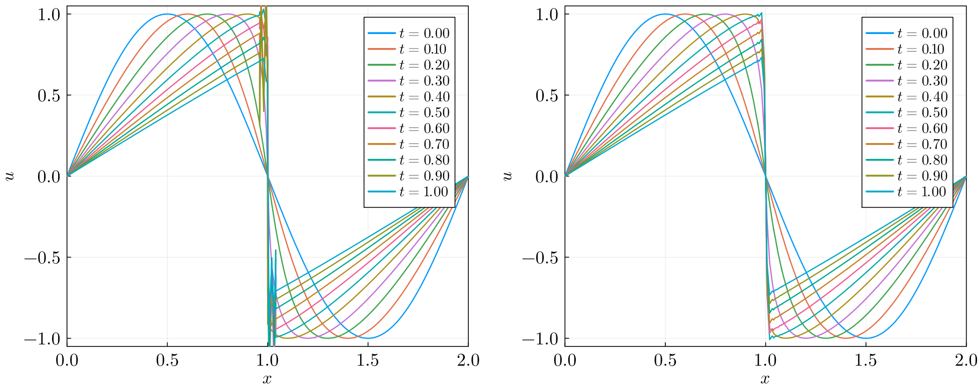

Figure 1: Comparison of the solution of the Burgers equation on a sine wave using p4 and 50 elements. Left: GLL nodes and Roe solver without entropy viscosity. Right: GLL nodes and Roe solver with entropy viscosity ($c_E = 1.0$).

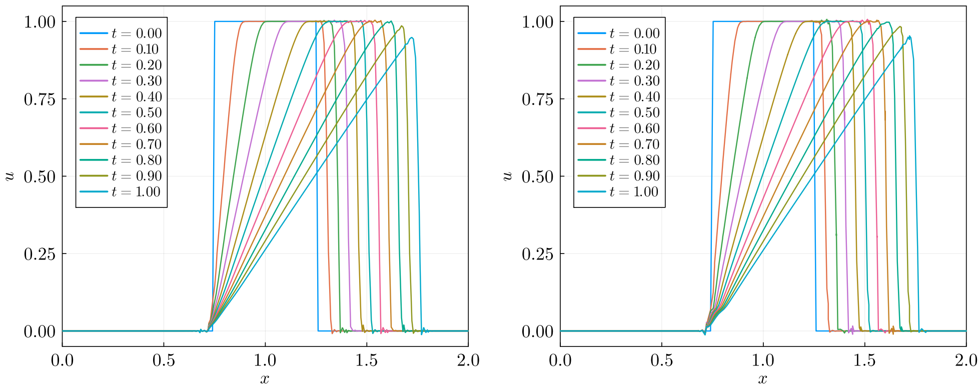

Figure 2: Comparison of the solution of the Burgers equation on a square wave using p4 and 50 elements. In both experiments the entropy viscosity is activated ($c_E = 1.0$) and a Roe solver is used at the interface. Left: GL quadrature nodes. Right: GLL quadrature nodes.

viscosity is needed to stabilize the shock when using GLL quadrature nodes and a Roe solver, as shown in fig. 1. For both quadratures points (GL results not shown here), oscillations are reduced on the shock interface without completely eliminating them, and the shock is better captured with increasing polynomial degree. For the square wave, even with a Roe solver on the element interface, the method is not able to capture the shock and the solution is very degraded or non-existent. The addition of the entropy viscosity allows to correctly capture the shock, as shown in fig. 2, although some dissipation is present since the shape of the wave is not completely recovered. With this encouraging results, the next step is to test this method in the well-known Sod shock problem for the 1D Euler equations. Moreover, split formulations of the governing equations will be explored as a de-aliasing mechanism (Abe, 2018). This will allow to further assess the validity of the entropy-viscosity shock-capturing scheme for the FR method.

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